CAT

Certified Accounting Technicians Examination

Stage: Level 1 L1.4

Subject Title: Business Mathematics

Revision Guide

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STUDY TECHNIQUE

What is the best way to manage my time?

- **Identify all available free time between now and the examinations.**

- **Prepare a revision timetable with a list of "*must do*" activities.**

- **Remember to take a break (approx 10 minutes) after periods of intense study.**

What areas should I revise?

- **Rank your competence from Low to Medium to High for each topic.**

- **Allocate the least amount of time to topics ranked as high.**

- **Allocate between 25% - 50% of time for medium competence.**

- **Allocate up to 50% of time for low competence.**

How do I prevent myself veering off-track?

- **Introduce variety to your revision schedule.**

- **Change from one subject to another during the course of the day.**

- **Stick to your revision timetable to avoid spending too much time on one topic.**

Are study groups a good idea?

- **Yes, great learning happens in groups.**

- **Organise a study group with 4 – 6 people.**

- **Invite classmates of different strengths so that you can learn from one another.**

- **Share your notes to identify any gaps.**

EXAMINATION TECHNIQUES

INTRODUCTION

Solving and dealing with problems is an essential part of learning, thinking and intelligence. A career in accounting will require you to deal with many problems.

In order to prepare you for this important task, professional accounting bodies are placing greater emphasis on problem solving as part of their examination process.

In exams, some problems we face are relatively straightforward, and you will be able to deal with them directly and quickly. However, some issues are more complex and you will need to work around the problem before you can either solve it or deal with it in some other way.

The purpose of this article is to help students to deal with problems in an exam setting. To achieve this, the remaining parts of the article contain the following sections:

- Preliminary issues

- An approach to dealing with and solving problems

- Conclusion.

Preliminaries

The first problem that you must deal with is your reaction to exam questions.

When presented with an exam paper, most students will quickly read through the questions and then many will … **PANIC!**

Assuming that you have done a reasonable amount of work beforehand, you shouldn't be overly concerned about this reaction. It is both natural and essential. It is natural to panic in stressful situations because that is how the brain is programmed.

Archaeologists have estimated that humans have inhabited earth for over 200,000 years. For most of this time, we have been hunters, gatherers and protectors.

In order to survive on this planet we had to be good at spotting unusual items, because any strange occurrence in our immediate vicinity probably meant the presence of danger. The brain's natural reaction to sensing any extraordinary item is to prepare the body for 'fight or flight'. Unfortunately, neither reaction is appropriate in an exam setting.

The good news is that if you have spotted something unusual in the exam question, you have completed the first step in dealing with the problem: its identification. Students may wish to use various relaxation techniques in order to control the effects of the brain's extreme reaction to the unforeseen items that will occur in all examination questions.

However, you should also be reassured that once you have identified the unusual item, you can now prepare yourself for dealing with this, and other problems, contained in the exam paper.

A Suggested Approach for Solving and Dealing with Problems in Exams.

The main stages in the suggested approach are:

1. Identify the Problem
2. Define the Problem
3. Find and Implement a Solution
4. Review

1. Identify the Problem

As discussed in the previous section, there is a natural tendency to panic when faced with unusual items. We suggest the following approach for the preliminary stage of solving and dealing with problems in exams:

Scan through the exam question

You should expect to find problem areas and that your body will react to these items.

PANIC!!

Remember that this is both natural and essential.

Pause

Take deep breaths or whatever it takes to help your mind and body to calm down.

Try not to exhale too loudly – you will only distract other students!

Do something practical

Look at the question requirements.

Note the items that are essential and are worth the most marks.

Start your solution by neatly putting in the question number and labelling each part of your answer in accordance with the stated requirements.

Actively reread the question

Underline (or highlight) important items that refer to the question requirements. Tick or otherwise indicate the issues that you are familiar with. Put a circle around unusual items that will require further consideration.

2. Define the Problem

Having dealt with the preliminary issues outlined above, you have already made a good start by identifying the problem areas. Before you attempt to solve the problem, you should make sure that the problem is properly defined. This may take only a few seconds, but will be time well spent. In order to make sure that the problem is properly defined you should refer back to the question requirements. This is worth repeating: Every year, Examiner Reports note that students fail to pass exams because they do not answer the question asked. Examiners have a marking scheme and they can only award marks for solutions that deal with the issues as stipulated in the question requirements. Anything else is a waste of time. After you have re-read the question requirements ask yourself these questions in relation to the problem areas that you have identified:

Is this item essential in order to answer the question?

Remember that occasionally, examiners will put 'red herrings' (irrelevant issues) into the question in order to test your knowledge of a topic.

What's it worth?

Figure out approximately how many marks the problem item is worth. This will help you to allocate the appropriate amount of time to this issue.

Can I break it down into smaller parts?

In many cases, significant problems can be broken down into its component parts. Some parts of the problem might be easy to solve.

Can I ignore this item (at least temporarily)?

Obviously, you don't want to do this very often, but it can be a useful strategy for problems that cannot be solved immediately.

Note that if you leave something out, you should leave space in the solution to put in the answer at a later stage. There are a number of possible advantages to be gained from this approach:

1) It will allow you to make progress and complete other parts of the question that you are familiar with. This means that you will gain marks rather than fretting over something that your mind is not ready to deal with yet.

2) As you are working on the tasks that you are familiar with, your mind will relax and you may remember how to deal with the problem area.

3) When you complete parts of the answer, it may become apparent how to fill in the missing pieces of information. Many accounting questions are like jigsaw puzzles: when

you put in some of the parts that fit together, it is easier to see where the missing pieces should go and what they look like.

3. Find and Implement a Solution

In many cases, after identifying and defining the problem, it will be easy to deal with the issue and to move on to the next part of the question. However, for complex problems that are worth significant marks, you will have to spend more time working on the issue in order to deal with the problem. When this happens, you should follow these steps:

Map out the problem

Depending on your preferred learning style, you can do this in a variety of ways including diagrams, tables, pictures, sentences, bullet points or any combination of methods. It is best to do this in a working on a separate page (not on the exam paper) because some of this work will earn marks. Neat and clearly referenced workings will illustrate to the examiner that you have a systematic approach to answering the question.

Summarise what you know about the problem

Make sure that this is brief and that it relates to the question requirements. Put this information into the working where you have mapped out the problem. Be succinct and relevant. The information can be based on data contained in the question and your own knowledge and experience. Don't spend too long at this stage, but complete your workings as neatly as possible because this will maximise the marks you will be awarded.

Consider alternative solutions

Review your workings and compare this information to the question requirements. Complete as much of the solution as you can. Make sure it is in the format as stipulated in the question requirements. Consider different ways of solving the problem and try to eliminate at least one alternative.

Implement a solution

Go with your instinct and write in your solution. Leave extra space on the page for a change of mind and/or supplementary information. Make sure the solution refers to your workings that have been numbered.

4. Review

After dealing with each problem and question, you should spend a short while reviewing your solution. The temptation is to rush onto the next question, but a few moments spent in

reviewing your solution can help you to gain many marks. There are three questions to ask yourself here:

Have I met the question requirements?

Yes, we have mentioned this already. Examiner Reports over the years advise that failure to follow the instructions provided in the question requirements is a significant factor in causing students to lose marks. For instance, easy marks can be gained by putting your answer in the correct format. This could be in the form of a report or memo or whatever is asked in the question. Likewise, look carefully at the time period requested. The standard accounting period is 12 months, but occasionally examiners will specify a different accounting period.

Is my solution reasonable?

Look at the figures in your solution. How do they compare relative to the size of the figures provided in the question?

For example, if Revenue were 750,000 and your Net Profit figure was more than 1 million, then clearly this is worth checking.

If there were some extraordinary events it is possible for this to be correct, but more than likely, you have misread a figure from your calculator. Likewise, the depreciation expense should be a fraction of the value of the fixed assets.

What have I learned?

Very often in exams, different parts of the solution are interlinked. An answer from one of your workings can frequently be used in another part of the solution. The method used to figure out an answer may also be applicable to other parts of your solution.

Conclusion

In order to pass your exams you will have to solve many problems. The first problem to overcome is your reaction to unusual items. You must expect problems to arise in exams and be prepared to deal with them in a systematic manner. John Foster Dulles, a former US Secretary of State noted that: *The measure of success is not whether you have a tough problem to deal with, but whether it is the same problem you had last year.* We hope that, by applying the principles outlined in this article, you will be successful in your examinations and that you can move on to solve and deal with new problems.

Stage: Level 1 L1.4
Subject Title: Business Mathematics

Examination Duration: 3 Hours

Assessment Strategy

Examination Approach
This subject deals with the collection and organisation of key business facts into meaningful data and the presentation and analysis of this data into useful information. Questions are framed in a business context, with each question having a number of sub-sections. The first sub-section may require quantitative analysis, while others may require qualitative analysis, that is, to provide an interpretation of the quantitative data.

Examination Format
The examination is unseen, closed book and 3 hours' in duration. Students are required to answer 5 questions out of 6.

Marks Allocation
Each question carries 20 marks The total for the paper is 100 marks.

Learning Resources
Core Texts
Curwin and Slater / Quantitative Methods for Business Decisions, 6th Edition / Cengage (2008) ISBN 1844805743
Clare Morris / Quantitative Approaches in Business Studies, 7th Edition / Pearson Education (2010) ISBN 027379417
Donald Waters / Quantitative Methods for Business, 4th Edition / Pearson Education (2007) ISBN 0273694588.
For greater depth on statistics, the following is recommended:
W.M. Harper / Statistics, 6th Edition / Pearson Education / ISBN 0273634267.

Manuals
Institute of Certified Public Accountants of Rwanda – L1.4 Business Mathematics

Supplementary Texts and Journals

Les Oakshott / Essential Quantitative Methods for Business, Management and Finance, 4th Edition/ /Palgrave (2009)/ ISBN 9780230218185.
Lucy T. / Quantitative Techniques 6th ed. / Continuum Publications (2002) / ISBN 0826458548.
Wisniewski M. / Quantitative Methods for Decision Makers 5th ed / Pearson 2009 / ISBN 9780273712077 / ISBN 0273712071.
Buglear J. / Quantitative Methods for Business Elsevier (2004) / ISBN 0750658983.
Buglear J. / Stats Means Business / Butterworth Heinemann (2001) / ISBN 0750653647.
Soper J. / Mathematics for Economics and Business / Blackwell (2004) / ISBN 1405111275.

Useful Websites

(as at date of publication)
http://www.icparwanda.com/services.php
http://ubalt.edu/ntsbarsh/Businessstat/ opre504.htm - Professor Hossein Arsham's, (FOR, FRSS, FWIF), Statistical Thinking for Managerial Decisions.

REVISION QUESTIONS AND SOLUTIONS

Stage: **Level L1.4**

Subject Title: **Business Mathematics**

QUESTION 1

Two companies have provided tenders for the replacement of production machines for DIY Ltd. Machine 1 will cost RWF25,000 and Machine 2 will cost RWF20,000. The financial accountant estimates that the other cash flows for the machines over their four year lifetime will be:

Cash inflows (RWF)

	Year 1	Year 2	Year 3	Year 4
Machine 1	18,000	22,000	12,000	8,500
Machine 2	10,000	12,000	16,000	17,000

Cash outflows (RWF)

	Year 1	Year 2	Year 3	Year 4
Machine	9,000	12,000	3,000	3,000
Machine 2	7,000	10,000	3,500	3,500

The loan rate charged by the bank is 14%. You may assume that the chosen machine is paid for upon delivery on site (purchased now) and the other cash flows will occur at the end of the year.

REQUIRED:

  a) Set out the net cash flows for each machine. **(4 Marks)**
  b) Derive the Net Present Value for each machine. **(6 Marks)**
  c) Based on your analysis advise the company which machine to purchase.

(4 Marks)

d) The supplier of Machine 1 has now offered to modify its payment terms. Instead of requiring an initial payment of RWF25,000, they will accept an initial payment of RWF15,000 with the balance of RWF10,000 payable in year 1. Which machine would you now advise the company to purchase? Support your answer with relevant calculations.

(6 Marks)
(Total: 20 Marks)

QUESTION 2

To demonstrate that it is providing loans to small companies, the GV bank provided the following data. This data summarises the number and level of loans to companies in both Kigali and Butare.

Loans (RWF000s)	Number of Companies	
	Kigali	**Butare**
24 < 28	8	19
28 < 32	41	36
32 < 36	77	47
36 < 40	90	58
40 < 44	58	27
44 < 48	26	13

REQUIRED:

a) Compare the mean loan and standard deviation for both Kigali and Butare.

(10 marks)

b) Derive the co-efficient of variation for both regions. **(6 marks)**

c) Explain the principles underpinning the calculations in (a) and (b) above.

(4 marks)
(Total: 20 Marks)

QUESTION 3

The local supermarket recently received a consignment of cereals from the national distributor. The management accountant believed that the consignment is incorrectly packed and randomly selected 49 bags to be sampled. The mean weight was found to be 42.4kgs with a standard deviation of 4kgs. According to the distributor, the bags should have had a mean weight of 40kgs.

REQUIRED:

a) Calculate the range of weights with a 95% confidence level. **(10 Marks)**
b) Test if the population mean is greater than 40kg with a 95% confidence level.
(10 marks)
(Total: 20 Marks)

QUESTION 4

The SME company wishes to establish a relationship between the costs of production and the output of the company. The following data, on units of output and total costs, has been collected over the last eight quarters:

Quarter	Output (units)	Cost RWF
1	10,000	32,000
2	20,000	39,000
3	40,000	58,000
4	25,000	44,000
5	30,000	52,000
6	40,000	61,000
7	50,000	70,000
8	45,000	64,000

REQUIRED:

a) Using linear regression analysis, derive the relationship between the variables.

(10 Marks)

b) Interpret the equation in terms of both fixed and variable costs of production.

(6 Marks)

c) Forecast the costs that should be incurred at an output level of 55,000 units

(4 Marks)

(Total: 20 Marks)

QUESTION 5

a) Your company is purchasing a Systems 102 shredder for RWF12,000. Estimate the equal annual payments, if the machine is being purchased with a 5 year loan compounded annually at 14%.

(6 Marks)

b) You wish to purchase a forklift for your business. Your bank quotes you an annual percentage rate (APR) of interest of 15% on a loan of RWF20,000 over 4 years. Calculate the total interest paid. Assume that the interest is paid at the end of each year.

(6 Marks)

c) Your company's internal auditors have examined the debtors accounts. The data is normally distributed with a mean value of RWF3,000 and a standard deviation of RWF500. You consider that 5% of the accounts are 'doubtful'. In this context you are asked to determine the probability that, if an account is randomly chosen, its value will be between RWF3,200 and RWF3,500.

(8 Marks)
(Total: 20 Marks)

QUESTION 6

"Time series data can be used as a basis for estimating both trend and seasonal variation. Such estimates can then be used as a basis for forecasting."

In this context:

a) Outline the structure of a time series including trend, seasonal, cyclical and irregular components.

(8 Marks)

b) Describe the moving average method for smoothing a time series. **(6 Marks)**

c) Describe how the time series can be used for forecasting. **(6 Marks)**

(Total: 20 Marks)

QUESTION 7

CPAR Consultants Ltd. is undertaking an analysis of a proposal by Superior Products Ltd. The company is investing in a moulding machine at a cost of RWF125,000. The machine will last for 5 years and will be sold for scrap at the end of year 6 for RWF5,000. The machine will require regular annual maintenance. At the end of year 1 this will amount to RWF1,500 and will increase by 20% each year until the end of year 4. The revenue produced by the machine is estimated at RWF30,000 at the end of year 1 and this will increase by 10% at the end of each year over the life of the machine.

As a consultant with CPAR you are required to:

   i.   Tabulate the net cash flows for the machine. **(8 Marks)**
  ii.   Establish the Internal Rate of Return (IRR) on the investment. **(8 Marks)**
 iii.   Interpret your result if the overdraft rate provided by the bank is 15%.

(4 Marks)
(Total: 20 Marks)

QUESTION 8

DIB Ltd. has two divisions with employees doing similar types of work. The data below gives the distribution of earnings of a sample of employees from the two divisions.

Earnings/week **RWF**	Number of employees	
	Division 1	Division 2
400 – 420	11	12
420 – 440	20	30
440 – 460	35	50
460 – 480	40	60
480 – 500	25	40
500 – 520	10	35

As the management accountant to the company you are asked to:

i.   Calculate the average earnings and standard deviation for employees of both divisions
(12 Marks)

ii.   Determine if there is any significant difference between the earnings in both divisions. Test at the 1% significance level. **(8 Marks)**
(Total: 20 Marks)

QUESTION 9

Fado Ltd is a training company which pays a Government allowance to trainees during various stages of training. A recent notice by the Office of the Comptroller General has indicated that the company will be subject to an audit in the near future. You have compiled the following data from the company records.

Monthly Allowance	Number of Trainees
260-280	8
280-300	14
300-320	16
320-340	15
340-360	9
360-380	7
380-400	6
400-420	5

In order to prepare for the audit you have been requested by the financial controller to prepare a report for the Board. As part of your report you are requested to:

   i.   Present the data in a histogram.

(8 Marks)

  ii.   Derive the modal and median allowances paid to trainees.

(8 Marks)

 iii.   Explain your results.

(4 Marks)
(Total: 20 Marks)

QUESTION 10

A company is negotiating with Superior Products Ltd for the supply of batteries for its torches. Superior Products needs to plan its production to meet the needs of the torch company. It uses the company's quarterly sales figures over the past three years to forecast future demand.

Year	Q1	Q2	Q3	Q4
2007	350	285	197	390
2008	450	420	325	460
2009	550	530	415	615

Using this data:

    i.   Derive the trend line equation using linear regression analysis

(10 Marks)

   ii.   Illustrate the data on a diagram and forecast the demand for the four quarters of 2010

(10 Marks)
(Total: 20 Marks)

QUESTION 11

As an independent financial adviser you are presented with the following problems from clients

i. The SPP Credit Union has launched a new saving scheme for investors. If your client invests RWF10,000 now and RWF5,000 at the end of each year for the next 3 years, he will receive 10% in year 1 and 15% in years 2 and 3. Advise your client of the sum receivable at the end of year 3 and the overall return on the investment.

(6 Marks)

ii. Mr P. Orr has been offered an encashment value of RWF10,000 now by the Bank of Kigali on his endowment mortgage. However, he estimates that it will mature to RWF13,500 in 5 years time applying an interest rate of 4%. Advise Mr P.Orr on the best option.

(6 Marks)

iii. "Paul's", the local builder's provider, consults you regarding the weights of concrete provided to construction sites. He believes that the quantities supplied exceed the required amounts. He wants to ensure that only 5% of any orders of concrete exceed the standard order weight of 1,000kg. The specification of the automatic weighing machine states that orders weigh with a standard deviation of 10kg. Advise Paul on the level that the machines average should be set to, assuming a normal distribution. Provide a diagram to support your advice.

(8 Marks)
(Total: 20 Marks)

QUESTION 12

You are frequently asked by the financial director to make presentations to the management team on modern business concepts. Discuss the following for inclusion in your next presentation:

(i)	Structure of a time series.	**(5 Marks)**
(ii)	Investment Appraisal.	**(5 Marks)**
(iii)	Network Analysis.	**(5 Marks)**
(iv)	Measures of dispersion.	**(5 Marks)**
		(Total: 20 Marks)

SUGGESTED SOLUTIONS

SOLUTION 1

a) and   b) Cash flows are set out in the table below.

Year	Cash Inflows	Cash Outflows	Net Cash Flows	Discount Factor	Present Value
	-	**-2**	**(1) - (2)**	**-14%**	
0	---------	-25,000	-25,000	1.	-25,000
1	18,000	- 9,000	-9,000	0.877	7,893

Year					
2	22,000	-12,000	10,000	0.769	7,690
3	12,000	-3,000	9,000	0.675	6,075
4	8,50	-3,000	5,500	0.592	3,256
				Net Present Value	(86)

(2 Marks)

Machine 2

Machine 1

c) The Net Present Value for machine 1 is a net loss of RWF86 while machine 2 provides a positive Net Present Value of RWF599. On this basis machine 2 is more financially attractive and should be selected by the company.

However, the difference between the NPV for both machines is small. Machine 1 has larger cash inflows during the early years while machine 2 has larger cash inflows during the latter years. The basic difference between the two proposals is the initial cost. A more appropriate analysis is to derive the internal rate of return which will provide a % return of both proposals which can be compared with the rate provided by the bank.

Year	Cash Inflows (1)	Cash Outflows (2)	Net Cash Flows (1) - (2)	Discount Factor (14%)	Present Value
0	---------	(20,000)	(20,000)	1.000	(20,000)
1	10,000	(7,000)	3,000	0.877	2,631
2	12,000	(10,000)	2,000	0.769	1,538
3	16,000	(3,500)	12,500	0.675	8,438
4	17,000	(3,500)	13,500	0.592	7,992
				Net Present Value	599

(3 Marks)

(4 Marks)

d) Based on the offer of phased payments for Machine 1, the revised Net Present Value for Machine 1 is a positive RWF1,144. Although Machine 2 also provides a positive Net Present Value of RWF599, it is less positive than Machine 1. On this basis Machine 1 is the more financially attractive and should be selected by the company.

Year	Cash Inflows	Cash Outflows	Net Cash Flows	Discount Factor	Present Value
	-	-2	(1) - (2)	-14%	

0	---------	-15,000	-15,000	1	-15,000
1	18,000	-19,000	-	0.877	-877
2	22,000	-12,000	10,000	0.769	7,690
3	12,000	-3,000	9,0	0.675	6,075
4	8,50	-3,000	5,5	0.592	3,256
				Net Present Value	**1,144**

(4 Marks)

(2 Marks)
(Total: 20 Marks)

SOLUTION 2

a) The mean loans and standard deviations are calculated below.

Kigali Region

Lower Boundary	Upper Boundary	Midpoint (x)	No companies (f)	fx	(x-x)	f(x̄-x)²
24	28	266	8	208	-11.03	973.29
28	32	30	41	1230	- 7.03	2026.26
32	36	34	77	2618	- 2.03	156.31
36	40	38	90	3420	0.97	84.68
40	44	42	58	2436	4.97	1432.60
44	48	46	26	1196	8.97	2091.96
∑			**300**			**6765.10**

Mean, $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{11,108}{300}$ = RWF37.03   **(2 marks)**

Standard deviation, $\sigma = \sqrt{\dfrac{\sum f(x-x)^2}{\sum f}}$ $= \dfrac{\sqrt{6765.1}}{300} = \sqrt{22.55} = 4.75$ **(3 marks)**

Butare Region.

Lower Boundary	Upper Boundary	Midpoint (x)	No companies (f)	Fx	(x-x)	f(x-x)²
24	28	26	19	494	-9.54	1729.22
28	32	30	36	1080	- 5.54	1104.89
32	36	34	47	1598	- 1.54	111.47
36	40	38	58	2204	2.46	350.99
40	44	42	27	1134	6.46	1126.75
44	48	46	13	598	10.46	1422.35
∑			**200**	**7108**		**5845.67**

Mean, $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{7,108}{200}$ = RWF35.54   **(2 marks)**

Standard deviation, $\sigma = \sqrt{\dfrac{\sum f(x - \bar{x})2}{\sum f}}$ $= \sqrt{\dfrac{5845.7}{200}} = \sqrt{29.22} = 5.40$ (3 marks)

b) Co-efficient of variation. This measures the relative dispersion between distributions and is derived from $\dfrac{\sigma}{x}$.

Using this co-efficient gives:

Kigali: $\dfrac{4.75}{37.03}$ x 100 = 12.83%;   **Butare**: $\dfrac{5.40}{35.54}$ x 100 = 15.19%

                                    (3 marks)   **(3 marks)**

c) From observations of the data from both series it is clear that the relative dispersion between the distributions is not wide. This calculation is more appropriate where there is substantial difference between the means of the distributions. The co-efficient is a measure that enables us to quantify the relative dispersion.

(4 marks)
(Total: 20 Marks)

SOLUTION 3

a) Establish the hypothesis that the population mean weight, μ, is 40kgs, that is, the Null Hypothesis (H_O). This means that we are assuming that there is no difference between the mean and the sample mean and that any difference can be ascribed to chance. The Alternative Hypothesis (H_1) can be stated that the population mean does not equal 40kg.

$$H_O: \mu = 40kg$$

$$H_1: \mu \neq 40kg$$

(2 marks)

If H_O is found to be true H_1 is rejected while if H_O is found to be false, H_1 is accepted. With a 95% confidence level the sample mean must lie within ± 1.96 standard errors (the acceptance zone), so that, if H_O is true, 95% of the means of all samples must be within the range $\mu \pm 1.96sx$ where $s\bar{x}$ is the standard error, that is, $s\bar{x} = \sigma/\sqrt{n}$.

(2 marks)

Therefore, $s\bar{x} = 4/\sqrt{49} = 0.57$.

The range is $40 \pm 1.96 \times 0.57$, that is, 38.88kgs to 41.12kgs **(2 marks)**

or the z-score $[(x - \mu)/s\bar{x} = (42.4 - 40)/0.57 = 4.21]$ must fall outside ± 1.96

As the sample mean of 42.4kg falls outside the acceptance zone the null hypothesis is rejected and H_1 is accepted, that is, $\mu \neq 40kg$. The difference between the population and sample means is significant at the 5% level. **(4 marks)**

b) In this test it is necessary to test if the population mean is greater than the assumed value of 40kgs, that is,

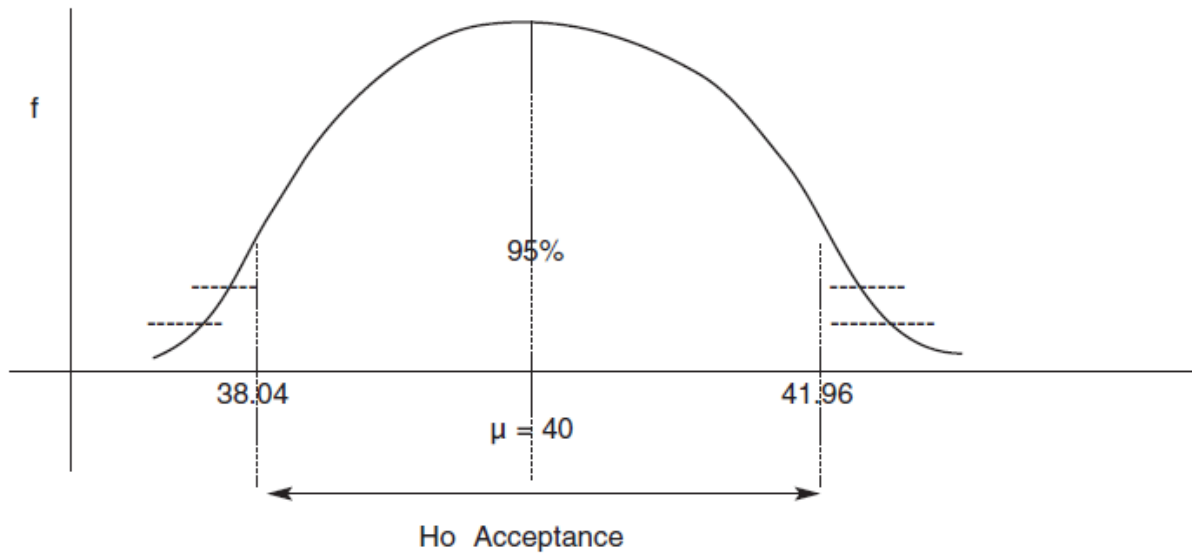H_o: μ = 40kgs; H_1: $\mu > 40$kgs. **(4 marks)**

This is a one tailed test where the number of standard errors at the 5% level is 1.65. Since the calculated z score is 4.21 and is outside the 5% value the Null Hypothesis is rejected.

(2 marks)

It can therefore be stated that the population mean is greater than 40kgs.

(2 marks)

The analysis is shown in the diagram for a two tailed test.

(2 marks)
(Total: 20 Marks)

SOLUTION 4

Quarter	Output (units)	Total Cost RWF
1	10,000	32,000
2	20,000	39,000
3	40,000	58,000
4	25,000	44,000
5	30,000	52,000
6	40,000	61,000
7	50,000	70,000
8	45,000	64,000

To determine the relationship between the variables, and the regression line $Y = a + bX$, we solve the following equations

$\sum Y = na + b\sum X$

$\sum XY = a\sum X + b\sum X2$ or   substitute the above values into the following   **(2 Marks)**

$b = \dfrac{\sum XY - \dfrac{\sum X \times \sum Y}{n}}{\sum X^2 - \dfrac{(\sum X)^2}{n}}$

$a = \dfrac{\sum Y}{n} - \dfrac{b\sum X}{n}$

X (000's)	Y (000's)	XY	X
10	32	320	100
20	39	750	400
40	58	2320	1600
25	44	1100	625
30	52	1560	900
40	61	2440	1600
50	70	3500	2500
45	64	2880	2025
260	420	14900	9750

Therefore, $\quad$ b $\quad = \dfrac{14900 - 260 \times 420/8}{9500 - (260)/8} = \dfrac{14900 - 13650}{9750 - 8450}$

$= 1250/1300$

$= 0.9615$ $\hfill$ **(2 Marks)**

a $\quad = 420/8 - 0.9615 \times 260/8$

$= 52.5 - 31.25$

$= 21.25$ $\hfill$ **(2 Marks)**

Y $\quad = 21{,}250 + 0.962X$ $\hfill$ **(4 Marks)**

b) Total Cost = Fixed cost + Variable cost per unit of production.   **(2 marks)**

In the above equation the fixed cost is RWF21,250; this cost is independent of the level of production. X represents the variable cost of production i.e. for each unit increase in production the costs increases by a factor of 0.962. At some stage during the production process the variable cost profile will intersect the fixed cost to give the breakeven value. The sum of the fixed and variable costs gives the total cost profile.

(4 marks)

c) The projected cost at 55,000 units: At output levels of 55,000 units, the likely incurred costs are:

$$Y \quad = \quad RWF21,250 + 0.962X \ 1,250$$

$$= \quad RWF21,250 \quad + 0.962 \ (55,000) \ = \ RWF52,910$$

(2 marks)

This projection is outside the boundaries of the relationship and the accuracy is predicated on the principle that the linear relationship will continue. On the assumption that the linearity of the relationship continues, the extrapolated figure can be accepted as being relatively accurate.

(2 marks)
(Total: 20 Marks)

SOLUTION 5

a) In this case we are talking about an amortisation annuity over the 5 years, that is, the amount borrowed (RWF12,000) is equal to the present value of the annual payments (say A) over the 5 years at the interest rate of 14%.

$$RWF12,000 = \frac{A}{1.14} + \frac{A}{(1.14)^2} + \frac{A}{(1.14)^3} + \frac{A}{(1.14)^4} + \frac{A}{(1.14)^5}$$

(3 Marks)

$$= A(0.8772 + 0.7695 + 0.6749 + 0.5921 + 0.5194)$$

$$= A(3.4331)$$

Therefore, $A = \dfrac{RWF12,000}{3.4331} = RWF3,495.$ **(3 Marks)**

b) The interest will be compounded over the four years. The compound interest is derived as follows (the total cost at the end of four years less the initial cost equals the interest paid)

$CI = P(1 + i)t - P,$ where $I = 15\%$, $P = RWF20,000$, $t = 4$.   **(3 Marks)**

Therefore, CI $=$ $20,000(1 + 0.15)^4$ $- 20,000$

$=$ $20,000 \times 1.749 - 20,000$

$=$ $34,980 - 20,000$ $= RWF14,980$

(3 Marks)

c) Probability of more than RWF3,200 is given by

Z $=$ $(3200 - 3000)/500$

$=$ 0.4

The probability is 0.3446 from the Normal distribution tables.   **(3 Marks)**

Probability of more than RWF3500 is given by

$$Z \quad = \quad (3500 - 3000)/500$$

$$= \quad 1.00$$

The probability is 0.1587 from the Normal distribution tables.   **(3 Marks)**

Therefore, probability is within the range 3200 to 3500, that is, 0.3446 – 0.1587, i.e. 0.1859.

(2 Marks)
(Total: 20 Marks)

SOLUTION 6

    a) Structure of a time series.

A time is a set of results for a particular variable of interest taken over a period of time. There are four separate elements to consider

- Trend component: in a set of time series data the measurements are taken at regular intervals. However, there may be random fluctuations in the data but in some cases the data will show a shift to lower or higher values over the time period in question. This movement is called the trend and is usually the result of some long term factors such as changes in expenditure, output, etc. There may be various trend patterns such as increasing linear trends, decreasing linear trend, non-linear trend or no trend.

(2 Marks)

- Cyclical component: a time series may show a trend of some sort but may also show a cyclical pattern of alternative sequences of observation above and below the trend line. This moves from the medium to longer term around the trend line. Any pattern of this type which lasts longer than a year is called a cyclical element for the time series. This is generally a very common occurrence.

(2 Marks)

- Seasonal component: there may be a pattern of variability in regular cycles within a year. The element of the time series which represents variability due to seasonal influences is called the seasonal element. It normally refers to movements over a one year period but it can also refer to any repeating pattern of less than one-year's duration

(2 Marks)

- Irregular component: this is the element which cannot be explained by the trend, cyclical and/or seasonal elements and is entirely unpredictable. It represents the random variability in the time series, caused by unanticipated and non-recurring factors which are unpredictable. It gives a dramatic and unexpected departure from trend.

(2 Marks)

b) Use of the moving average method for smoothing.

Smoothing methods are used to smooth out the irregular element of time series where there are no significant trends, cyclical or seasonal pattern.

Moving average: this method involves using the average of the most recent data values to forecast the next period. The number of data values used can be selected in order to minimise the forecasting error. To find the moving average, the simple average for a specified number of items of data is found. For example, with quarterly data a simple average for four items of data can be calculated and then move that average along. This is done by recalculating the average having dropped the initial item of data and adding a subsequent item of data. This gives the four quarter moving total. The moving totals fall in between the actual quarterly data, that is, the first moving total falls between quarters 2 and 3. The respective four quarter moving totals are divided by 4 to find the four quarter moving average. The data is centred by summing respective pairs of data and dividing by two. This smooths the data so that the trend line can be derived. In this way moving averages are used to 'smooth out' fluctuations in the data and obtain a trend line. This is the moving average trend which can be plotted to show the direction. Since the trend does not vary significantly in short time periods the trend can be projected to forecast future events.

However, there are limitations to the use of moving averages. Equal weighting is given to each of the values used in the moving average calculation whereas the most recent data is more relevant to current conditions. The moving average takes no account of data outside the period of average so full use is not made of all the data. The use of the unadjusted moving average as a forecast can cause misleading results when there is an underlying seasonal variation.

(6 Marks)

c) Use of a time series for forecasting.

Forecasting from the time series lies in our knowledge of the behaviour of trend. Since the trend does not vary significantly in short time periods, the trend can be projected to forecast future events. A time series can be used to forecast where there are no significant trends, cyclical or seasonal elements using the moving averages method as set out above. Using moving averages, to get an accurate projection of trend without plotting the values, it is necessary to derive the average rate of change of the trend. This can be done by deducting the first value of the trend from the last trend value, taking the average (less one) and adding to the last trend value. This can be done for a number of values to forecast for a number of

periods into the future. This is the moving average trend which can be plotted to show the direction. The trend can be projected forward. However, the further the trend is projected the greater is the possibility of inaccuracy.

However, if there are both trend and seasonal elements in the series there are other methods which may be used. The first stage in such a forecasting technique is to calculate the seasonal factors by smoothing out the time series using the moving averages method. If the seasonal factors are quarterly then the moving average is calculate on groups of four data points. When seasonal elements are involved it is necessary to work out the centred moving averages which are averages of the moving averages. Dividing the original observation by the equivalent centred moving average gives a seasonal factor for each observation. This is done for all quarters.

If the time series shows a long term linear trend, linear regression may be used. A linear regression equation linking dependent and independent variables can be derived but this assumes that we have a linear trend and that the future will be like the past.

(6 Marks)
(Total: 20 Marks)

SOLUTION 7

1. (i) Cash Flows for the proposal.

Year End	Machine Cost RWF	Maintenance Cost RWF	Revenue	Net Cash Flows RWF
0	125,000			(125,000)
1		(1,500)	30,000	28,500
2		(1,800)	33,000	31,200
3		(2,160)	36,300	34,140
4		(2,592)	39,930	37,338
5			43,923	43,923
6	5,000			5,000
	(2 Marks)	**(2 Marks)**	**(2 Marks)**	**(2 Marks)**

(ii)   Calculation of the Internal Rate of Return. To calculate the IRR it is necessary to derive present values. Taking discount values at 14% and 20% the following table is developed.

Year	Net Cash Flows RWF	Discount Factor @ 12%	PV RWF	Discount Factor @ 18%	PV RWF
0	(125,000)	1.000	(125,000)	1.000	(125,000)
1	28,500	0.893	25,450.5	0.847	24,139.5
2	31,200	0.797	24,866.4	0.718	22,401.6
3	34,140	0.712	24,307.7	0.609	20,791.3
4	37,338	0.636	23,747.0	0.516	19,266.4
5	43,923	0.567	24,904.3	0.437	19,194.4
6	5,000	0.507	2,535.0	0.370	1,850.0
NPV			810.9		(17,356)
			(2 Marks)		**(2 Marks)**

The IRR can be derived by calculation or graphically. Using the formula

$$\frac{N_1 I_2 \quad - \quad N_2 I_1}{N_1 \quad - \quad N_2}, \text{ where discount rate } I_1 \text{ gives NPV } N_1 \text{ and discount rate } I_2 \text{ gives NPV } N_2$$

$N_1 = RWF810.9, \quad I_1 = 12\%; \quad N_2 = (RWF17,356), \quad I_2 = 18\%$

$$IRR = \frac{810.9 \times 0.18 - (17,356) \times 0.12}{810.9 - (17,356)} = \frac{2,229}{18,167} = 12.3\%$$

(4 Marks)

(iii)　Interpretation of the result.　The value of 12.2% can be interpreted as the rate of return that the project will earn. On face value the NPV is positive at 12% and this represents a positive result. However, if the cost of capital to the company, represented by the rate of interest charged by the bank of 15%, is greater than this value, the project is not viable. However, there are mechanisms for deriving the real cost of capital to the company based on its equity and funds which may not be representative of the bank rates. The decision of the company in this case is that the project, as a stand alone investment, will not contribute the required return.

(4 Marks)
(Total: 20 Marks)

SOLUTION 8

i. The average earnings and standard deviations for employees in both divisions.

Division 1

Lower Boundary	Upper Boundary	Freqf	Mid point (x)	Fx	fx2
400	420	11	410	4,510	1,849,100
420	440	20	430	8,600	3,698,000
440	460	35	450	15,750	7,087,500
460	480	40	470	18,800	8,836,000
480	500	25	490	12,250	6,002,500
500	520	10	510	5,100	2,601,000
	$\sum$	141		65,010	30,074,100

Mean = x = $\dfrac{\sum fx}{\sum f}$ = $\dfrac{65,010}{141}$ = RWF461.1   **(3 Marks)**

Standard Deviation

$$\sigma = \sqrt{\dfrac{\sum fx^2}{\sum f} - \left\{\dfrac{\sum fx}{\sum f}\right\}^2} = \dfrac{\sqrt{3,074,100 - 212,578}}{141}$$

= $\sqrt{731}$ =   RWF26.71   **(3 Marks)**

Division 2

Lower Boundary	Upper Boundary	Freqf	Mid point (x)	Fx	fx^2
400	420	12	410	4,920	2,017,200
420	440	30	430	12,900	5,547,000
440	460	50	450	22,500	10,125,000
460	480	60	470	28,200	13,254,000
480	500	40	490	19,600	9,604,000
500	520	35	510	17,850	9103,500
	$\sum$	227		105,970	49,650,700

Mean = x = $\dfrac{\sum fx}{\sum f}$ = $\dfrac{105,970}{227}$ = RWF466.8   **(3 Marks)**

Standard Deviation.

σ = $\dfrac{\sqrt{\sum fx2 - \{\sum fx\}2}}{\sum f \quad \{\sum f\}}$ = $\dfrac{\sqrt{49,650,700 - 217,928}}{227}$

= $\sqrt{797}$ = RWF28.23   **(3 Marks)**

Summary

	Division 1	**Division 2**
Mean	RWF461.1	RWF466.8
Std Deviation	RWF26.71	RWF28.23
Number	141	227

ii. In order to test if there is a difference in the earnings between both divisions a hypothesis can be used. In this case we are testing for the difference of means in a similar way as confidence intervals for the difference of means. The difference between the sample estimates is first calculated and the values given by the null hypothesis, transform the difference to a number of standard errors, then compare with a critical value consistent with the alternative hypothesis. In this way we are comparing if the two sample means have come from the same population,

i.e. Null Hypothesis, H_O: $\mu_1 = \mu_2$ and the alternative hypothesis

H_1: $\mu_1 \neq \mu_2$ **(4 Marks)**

This is a two sided test. In this particular test we are assuming H_0 to be correct.

The Z statistic is $\dfrac{X1 - X2}{\sqrt{\dfrac{S1^2}{N_1} + \dfrac{S2^2}{N_2}}}$, that is, the difference in means divided by the standard error

$X_1 - X_2 = 461.1 - 466.8 = -5.7$

$\dfrac{S_1^2}{N_1} = \dfrac{26.71^2}{141} = 5.06;$ $\qquad$ $\dfrac{S_2^2}{N_2} = \dfrac{28.23^2}{227} = 3.51;$

$Z = \dfrac{-5.7}{\sqrt{8.57}} = -1.94$ **(4 Marks)**

Z, at 1% significance level, is 2.58.

Since the calculated value is less than this level we accept the null hypothesis – the samples do not provide evidence that there is a difference in the long-run earnings of employees.

(Total: 20 Marks)

SOLUTION 9

i.   Histogram of the data.

Monthly Allowance RWF	Number of Trainees (f)	Cumulative Frequency
260-280	8	8
280-300	14	22
300-320	16	38
320-340	15	53
340-360	9	62
360-380	7	69
380-400	6	75
400-420	5	80
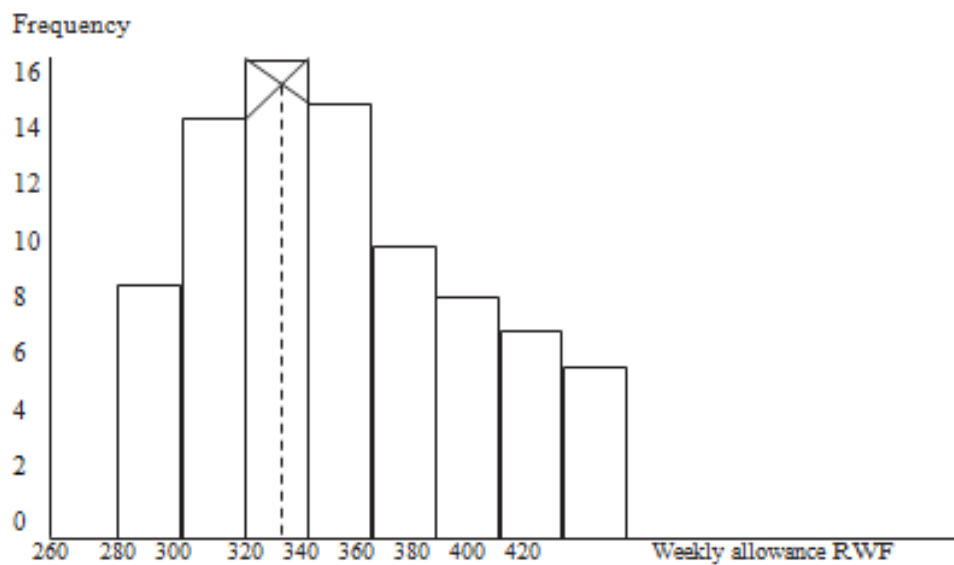
(8 Marks)

The modal value of the allowance is approximately RWF310, found from the histogram.

ii. The Median.

The median is the value of the 40.5 trainee in group 320 – 340.
From the formula the median value = 320 + (40.5 – 38)/15 x 20 = RWF323.33.

(6 Marks)

iii. Comparison of the mode and median.

The mode is the greatest frequency of the distribution and is in the range RWF320 - RWF340. The accurate value is derived from the histogram above.

The median represents 50% of the data above and below the mid point. It means that 50% of the trainees are receiving more than RWF323.23 and 50% less than RWF323.23

(8 Marks)

SOLUTION 10

i.   The trend for this data can be developed using either a moving average technique or
     linear regression analysis. A four quarter centred moving average could be derived
     allowing us to decompose the data to isolate the trend by means of either additive or
     multiplicative models. In the present case it may be substantially easier to develop the
     trend by means of linear regression without smoothing the data, that is,

$$y = a + bx \text{ where}$$

$$a = \frac{\Sigma y}{n} - \frac{b\Sigma x}{n}$$

$$b = \frac{\Sigma xy - (\Sigma x \Sigma y)/n,}{\Sigma x2 - (\Sigma x)2/n}$$

Year	Quarter x	Sales y	x^2	xy
1	1	350	1	350
	2	295	4	590
	3	197	9	591
	4	390	16	1,560
2	5	450	25	2,250
	6	420	36	2,520
	7	325	49	2,275
3	8	460	64	3,680
	9	550	81	4,950
	10	530	100	5,300
	11	415	121	4,565
Σ	12	615	144	7,380
	78	4,997	650	36,011

(5 Marks)

Inserting values gives

$$b = \frac{36,011 - (78 \times 4997)/12}{650 - 55^2/12}$$

$$= \frac{3530.5}{143} = 24.69$$
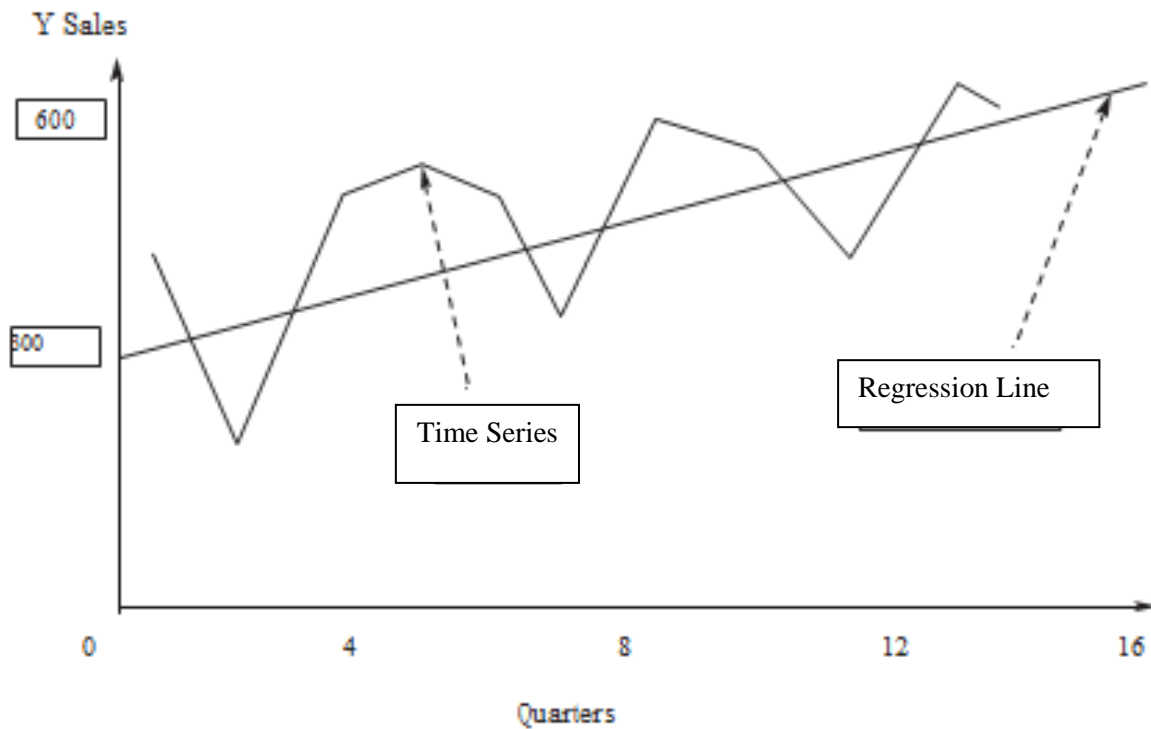
$$a = \frac{4997}{12} - \frac{24.69 \times 78}{12} = 416.4 - 160.5 = 255.9$$

Therefore, y = 255.9 + 24.69x.   **(5 Marks)**

ii.   Plotting a graph of the points and projecting the data for 2010.

(5 Marks)

The projected points for the four quarters of 2010 are calculated from the regression line for quarters

        13,14,15,16.

For x = 13, y (Sales) = 576,870

For x = 14, y (Sales) = 601,560

For x = 15, y (Sales) = 626, 250

For x = 16, y (Sales) = 650,940

(5 Marks)
(Total: 20 Marks)

SOLUTION 11

    i.   Initial Sum invested: RWF10,000

At end year 1

+10%	RWF 1,000	
	RWF 5,000	RWF16,000

At end year 2

+15%	**RWF 2,400**	
	RWF 5,000	**RWF23,400**

At end year 3

+15%	RWF 3,510	
	RWF 5,000	RWF31,910

                                      ---- Sum Receivable

(3 Marks)

Return on investment: $\dfrac{(RWF31,910 - RWF25,000) \times 100}{RWF25,000} = 27.64\%$

(3 Marks)

    ii.   Discounting RWF13,500 to present values at 4% gives $\dfrac{13,500}{1.04^5} = \dfrac{13,500}{1.2166}$

$$= RWF11,096$$

Since this value is greater than RWF10,000, the option to receive RWF13,500 in 5 years is the best option.

(6 Marks)

iii.   Standard deviation = 10kg; Mean = ?; x = 1000kg.

Converting these values to the standard Normal Distribution $z = \dfrac{x - \mu}{\sigma}$

For 5% (0.05), z = 1.645
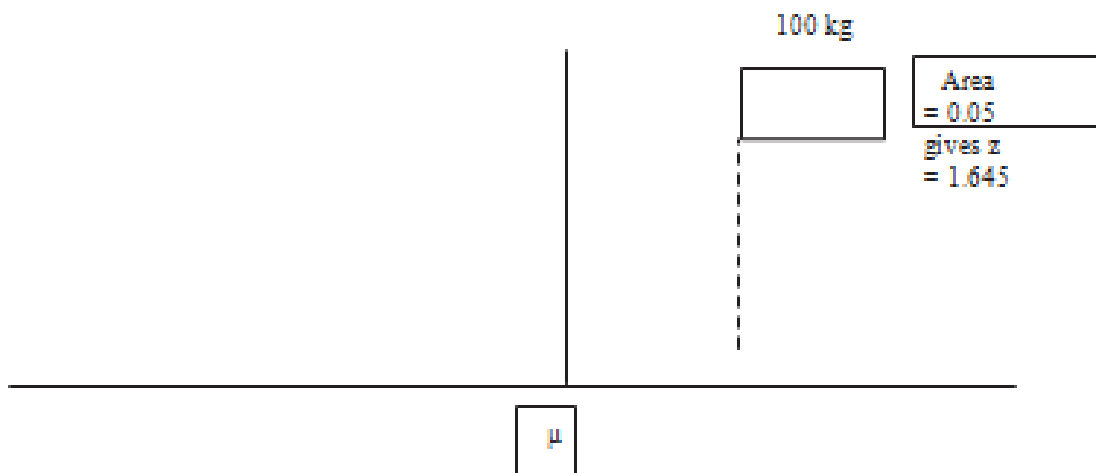
Therefore, $1.645 = \dfrac{1000 - \mu}{10}$

μ = 1000 - 1.645 x 10

= 983.55kg

To ensure that only 5% of the orders are overweight the machine should be set to 983.55kg.

(4 Marks)

Diagram

(4 Marks)

SOLUTION 12

An explanation of the terms is provided below.

i.   **Structure of a time series.** A time series is a set of results for a particular variable taken over a period of time. There are four separate elements in the structure

- **Trend**: In many cases the data exhibits a shift either to lower or higher values over the time period in question. This movement is called the trend and is usually the result of some long-term factors such as changes in expenditure, sales, demographic factors, etc. There may be many trend patterns – increasing linear trend, decreasing linear trend, non-linear trend or no trend.

- **Cyclical**: A time series may display a trend of some form but may also show a cyclical pattern of alternative sequences of observation above and below the trend line. Any regular pattern which lasts longer than a year is the cyclical element of the time series. This is a common occurrence represented by the retail sector of business.

- **Seasonal**: although the cyclical patter may be displayed over a number of years, there may be a pattern of variability within one-year periods. The element of the time series which represents variability due to seasonal influences is the seasonal element. While it normally refers to patterns over a one-year period, it can also refer to any repeating pattern of less than one-year's duration.

- **Irregular element**: this is the element which cannot be explained by the trend, the cyclical or seasonal elements. It represents the random variability in the time-series, caused by unanticipated and non-recurring factors which are unpredictable. There are particular methods used to smooth out the irregular element of time series where there are no significant trends, cyclical or seasonal patterns such as moving averages or exponential smoothing.

(5 Marks)

ii.   **Investment Appraisal**. Many use many quantitative techniques as an aid to decision making. One of the most important applications is concerned with investment decisions. Managers often have to make choices between alternative decisions, they need to consider future costs and revenues and the importance of incremental changes in costs and revenues. It is also critical to consider the time value of money because of

time scales involved. The manager's decision to invest is based on three important factors – the estimate of the future based on forecasts of costs, revenues, inflation, and interest rates; the alternatives available for investment: techniques to support the decision must be used; the business attitude to risk: because of the uncertainty of the future and project uncertainty, additional techniques must be used. A number of these techniques used in investment appraisal are, the accounting rate of return; payback; discounted cash flow, such as net present value and internal rate of return.

The accounting rate of return is the ratio of the average annual profits, after depreciation, to the capital involved. Variations of this exist but it has some drawbacks – it does not allow for the timing of cash flows and profit has subjective elements. Payback is a popular technique which is defined as the period which it takes for the projects net cash flows to recover the original investment. Payback favours quick return projects – the project with the shortest payback period is accepted. However, it does not measure overall investment worth since it does not consider cash flows after the payback period or the timing of cash flows. Net Present Value uses discounting principles and involves calculating the present value of expected cash inflows and outflows and establishing whether the present value of the net cash flows is positive. The internal rate of return is the discount rate which gives zero NPV. If the calculated IRR is greater than the company's cost of capital the investment is accepted. However these two techniques do not necessarily rank investment proposals in the same order of attractiveness. But the IRR technique is used extensively in investment appraisal decisions.

(5 Marks)

  iii.   **Network Analysis.** This deals with a range of techniques used to aid managers in the planning and control of projects. These techniques show the inter-relationship of the various jobs to be done to complete the project and clearly identify the critical parts of the project. They provide planning and control information on the time, cost and resources required. These techniques are of particular value where projects are large and contain many related and interdependent activities, where many types of facilities, high capital investments and a large number of staff are involved, where projects have to be completed within target time and costs.

- Path analysis. To develop the technique it is necessary to know the activities involved, their logical relationship, an estimate of the time the activity is expected to take. It may be necessary to have a range of estimates of times, costs, resources and probabilities. Once the logic of the activities has been agreed and an outline network drawn, it can be completed by inserting the activity duration times. These times may be estimated by using probabilities (developing the expected time) or using basic time

estimates. A basic feature of the analysis is the calculation of the project duration which is the duration of the critical path. This is the path through the network which gives the shortest time in which the whole project can be completed. It is the chain of activities with the longest duration times. In a network there may be more than one critical path. The activities along the critical path are vital activities which must be completed on time otherwise the project will be delayed. If it is required to reduce the overall project duration then the time of one or more of the activities on the critical path must be reduced.

A further important feature of the analysis deals with the process of cost scheduling. This implies that by using more resources the duration could be reduced but at the expense of higher costs. The costs associated with delivery of the project, as designed, is the 'normal' cost. It is possible to reduce the activities by means of extra wages, overtime, additional facility costs which are higher than normal and are called crash costs. This is an element of least cost scheduling which finds the least cost method of reducing the overall project duration, time period by time period.

(5 Marks)

iv. **Measures of dispersion**. Measures of dispersion represent a measure of the extent of spread of data around the mean or average of the data. The simplest measure of dispersion is to take the absolute difference between the highest and lowest value of the raw data – 'the range'. The interquartile range is the absolute difference between the upper and lower quartiles of a distribution. These measures of dispersion involve comparing two different points on the frequency distribution such as the maximum and minimum points (the range) or the upper and lower quartiles. However, other measures of dispersion which compare all the points on the frequency distributions are more important. The 'mean deviation', that is, the average of the absolute deviations from the arithmetic mean, may be used. However, if is the basis for the most common measures of dispersion – the variance and the standard deviation. The variance is derived by squaring all the deviations from the arithmetic mean. Rather than use squared units, it is more realistic to express solutions to problems in terms of single units. The standard deviation takes the square root of the variance. The standard deviation is then the square root of the average of the squared deviation from the mean and is the most commonly used measure to explain the dispersion of data. These various methods can be used for both grouped and ungrouped data.

(5 Marks)